Chaotic scattering in heavy–ion reactions

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Abstract

We discuss the relevance of chaotic scattering in heavy–ion reactions at energies around the Coulomb barrier. A model in two and three dimensions which takes into account rotational degrees of freedom is discussed both classically and quantum-mechanically. The typical chaotic features found in this description of heavy-ion collisions are connected with the anomalous behaviour of several experimental data.

I. INTRODUCTION

The study of classical dynamical chaos has been extended in the last years to the case of open systems. It has been found that scattering variables have an irregular behaviour as a function of the initial conditions when the interaction zone is chaotic. Though scattering trajectories explore the real chaotic region only for a finite time, their behaviour can be so complicated that the final observables show strong and unpredictable fluctuations. These fluctuations are present on all scales of the initial conditions, revealing an infinite set of singularities with a Cantor-like fractal structure. Singularities are connected with those trajectories that remain trapped in the interaction region for very long times. In this sense the term chaotic has been extended also to scattering situations. Many examples have been investigated [1–10] and the phenomenon is so wide-spread to make one think that it is the rule rather than the exception. Important consequences of the underlying classical chaoticity have been found also in the semiclassical and quantal scattering counterparts [3,4,6,11–15].

In this paper we discuss the occurrence of chaotic scattering in nuclear reactions. Investigations on chaotic motion in nuclear physics started long ago [16] and they have been further stimulated by the recent progress on dynamical systems [17–20]. Scattering experiments are one of the main tools to study the nuclear structure, therefore it is rather important to know their regular or chaotic character. In particular studies with heavy ions cover a wide area of interest due to the great variety of nuclear phenomena which can be investigated [21,22].

We consider in particular the reaction between a spherical and a deformed nucleus taking into account rotational degrees of freedom only. This is a simplified description of the way in which two nuclei can interact, but it can be considered very realistic for many heavy–ion reactions. We show that even a few degrees of freedom can produce a very complicated and unpredictable motion.

This subject has been already discussed in several published papers [8,12–14], however, in the following we review in a coherent and general way what has been found including new and more detailed results. At the same time we try to use a simple and schematic language

in order to explain even to the less expert reader the reason of chaoticity onset, the meaning of it and the experimental implications according to the present understanding.

Unlike other papers in the present focus issue, our point of view is more phenomenological in the sense that we consider realistic potentials using the actual units adopted in nuclear physics. On the other hand, in this field and in particular in heavy-ion scattering the knowledge of the parameters which define the ion-ion potential are known within a 10-20% uncertainty. Therefore it would be meaningless to investigate the peculiarities of the scattering related to the finest details of the potential. However, adopting the well developed techniques extensively used for very simple hamiltonians, it is demonstrated that chaotic scattering in heavy–ion reactions is ubiquitous and does not depend on these details. It is shown as well that chaotic scattering is not a far out possibility, having real and serious implications which can be found experimentally. We discuss in particular in section II and III the classical dynamics of a reaction between a spherical and a deformed nucleus both in two and three dimensions. Actually it is shown that the former is a particular case of the latter. In section IV the quantal dynamics is then studied by means of a coupled channel approach. The connection between classical and the quantal treatment is discussed in section V. Realistic quantal calculations are then presented in section VI. Finally the connection with real experiments is illustrated in section VII. A summary is done in section VIII.

II. CLASSICAL SCATTERING

First of all we introduce a three-dimensional model to describe the scattering between a spherical nucleus (1) and a deformed one (2). Using polar coordinates, the Hamiltonian depends on 5 degrees of freedoms, i.e. r, θ and ϕ to describe the motion of the spherical projectile and Θ, Φ for the deformed rotor, see fig.1. Thus the Hamiltonian can be written as

$$H = H(r, \theta, \phi, \Theta, \Phi) = T(r, \theta, \phi) + H_2(\Theta, \Phi) + V(r, \theta, \phi, \Theta, \Phi), \tag{1}$$

where T is the kinetic term, H_2 the Hamiltonian of the deformed nucleus 2 and V the interaction potential. More precisely, T is given by

$$T = \frac{p_r^2}{2m} + \frac{1}{2mr^2} \left(p_\theta^2 + \frac{p_\phi^2}{sin^2\theta} \right) , \tag{2}$$

being m the reduced mass and p_r, p_θ, p_ϕ the conjugate momenta of r, θ and ϕ, while H_2 is

$$H_2 = \frac{1}{2\Im} \left(p_\Theta^2 + \frac{p_\Phi^2}{sin^2\Theta} \right) . \tag{3}$$

In this equation $\Im$ indicates the moment of inertia, while p_Θ and p_Φ the conjugate momenta of Θ and Φ, respectively.

The ion-ion potential V contains the monopole and quadrupole term of the Coulomb interaction plus the nuclear part U_N

$$V = \frac{Z_1 Z_2 e^2}{r} + \frac{Z_1 Q_0 P_2(cos\xi)}{2r^3} + U_N(r,\xi) , \tag{4}$$

with

$$cos\xi = cos\Theta cos\theta + sin\Theta sin\theta cos(\Phi - \phi) , \tag{5}$$

being ξ the angle between the rotor symmetry axis and the line joining the centers of the two nuclei. The symbol Q_o indicates the intrinsic quadrupole moment, while P_2 is the Legendre polynomial of order 2. A similar Hamiltonian has already been used [23] to study a typical heavy-ion scattering. In our case, we have chosen as nuclear interaction the *proximity* potential [24,21]. The latter is extracted taking into account the interaction energy per unit area between two curved nuclear surfaces. This choice has nothing special and it has been considered only because this potential is one of the most commonly used for deformed nuclei. The formula of the proximity potential is

$$U_N(r,\xi) = 4 \pi b \gamma \, \Re \, \psi(s(\xi)) , \tag{6}$$

with the proximity universal function given by

$$\psi(s(\xi)) = \begin{cases} -\frac{1}{2} (s - 2.54)^2 - 0.0852 (s - 2.54)^3 & if \ \ s \leq 1.2511 \\ - 3.437 \ exp(-s/0.75) & if \ \ s > 1.2511 . \end{cases} \tag{7}$$

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The distance s between the two nuclear surfaces is

$$s(\xi) \;=\; \frac{r - R_1 - R_2(\xi)}{b} \;,$$ (8)

with the nuclear radii [21]

$$R_i^o = (1.28 A_i^{\frac{1}{3}} - 0.76 + 0.8 A_i^{-\frac{1}{3}}) \;\; fm \qquad i = 1, 2$$ (9)

$$R_2(\xi) \;=\; R_2^o \left(1 \; + \; \alpha_{20} \, Y_{20}(\xi) \right) \;,$$ (10)

and the quantity $\Re$ defined as

$$\Re(\xi) \;=\; \frac{R_1 R_2^o}{R_1 + R_2^o} \left(1 \; - \; \frac{2 R_1}{R_1 + R_2^o} \, \alpha_{20} \, Y_{20}(\xi) \right) \;.$$ (11)

In the latter α_{20} is the deformation parameter and Y_{20} the spherical harmonic of order 2. The quantity γ is the so-called surface tension and it is given by [21]

$$\gamma = 0.95 \left[1. - 1.8 \left(\frac{N_1 - Z_1}{A_1} \right) \left(\frac{N_2 - Z_2}{A_2} \right) \right] \;\; MeV \; fm^{-2} \;,$$ (12)

being N_i and A_i the neutron and atomic numbers of the two nuclei, while b is the diffuseness parameter which is equal to 1 fm. In the following the units which are used are those commonly adopted in nuclear physics, that is fm for distances and MeV for energies. The actual value of $\hbar$ has been considered, more precisely it has been used $\hbar \, c \; = \; 197.329 \; MeV \; fm$.

It should be noted that eq.(8) represents the coupling between the relative motion and the internal (rotational) degrees of freedom. This coupling, breaking the central symmetry of the potential, is the one responsible of the onset of chaotic scattering as discussed later.

Solving the equations of motion for the Hamiltonian (1) one can follow in time the evolution of the system. However these equations are very general and complicated, thus in order to show in a clear and simple way the typical features of chaotic motion, let us consider for the moment the scattering occurring on the x-y plane. In this case we have only 3 degrees of freedom, i.e. r, ξ and ϕ, and the Hamiltonian reduces to

$$\mathcal{H} = \frac{p_r^2}{2m} + \frac{I^2}{2 \Im} + \frac{\ell^2}{2 m r^2} + V(r, \xi) \;,$$ (13)

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where $\xi = \Phi - \phi$, and $\ell = L - I$ is the orbital angular momentum with L and I the total angular momentum and the spin of the rotor, respectively.

The equations of motion corresponding to the Hamiltonian $\mathcal{H}$ are therefore

$$
\begin{aligned}
\dot{r} &= \frac{p_r}{m} \\
\dot{p}_r &= \frac{(p_\phi - p_\xi)^2}{2mr^3} - \frac{\partial V(r,\xi)}{\partial r} \\
\dot{\xi} &= \frac{I}{\mathfrak{I}} - \frac{(L-I)}{mr^2} \\
\dot{I} &= \dot{p}_\xi &&= -\frac{\partial V(r,\xi)}{\partial \xi} \\
\dot{L} &= \dot{p}_\phi &&= 0
\end{aligned}
\tag{14}
$$

Our Hamiltonian has two constants of motion, namely the total energy E and the total angular momentum L, as it can be seen from the last of eqs.(14). Neglecting the ξ-dependence of the full ion-ion potential, the Hamiltonian is separable and thus integrable, because the internal angular momentum I and the orbital one ℓ are conserved separately. However the ξ-dependence of the ion-ion potential introduces a symmetry-breaking term leading to the conservation of L only and generating the onset of chaos. In reality the scattering problem is integrable asymptotically. It is the chaoticity of the interaction zone which makes the scattering become chaotic. The set of unstable phase space trajectories which are confined in the interaction region defines the so-called *repeller*. The latter has an unstable manifold which extends to asymptotic distances, thus scattering trajectories are trapped for long but finite times inside the phase space region. The erratic, though deterministic, motion of these trapped trajectories, which are those that come closest to the repeller, cause the unpredictability of the final scattering observables on all scales.

In the following we solve numerically eqs. (14) for the planar case in order to illustrate the regular or chaotic character of the nuclear scattering. As a first example we take into account the reaction between the ^{28}Si nucleus considered spherical and the deformed ^{24}Mg. The values adopted for the deformation parameter α_{20} and the quadrupole moment Q_o taken from ref. [25,26] are reported in table 1. The potential $V(r,\xi)$ is shown in fig.2 as a function of r. The dependence on the angle, for the cases $\xi = 0^o$ and 90°, is illustrated for three initial orbital angular momenta $\ell = 15, 35, 45\,\hbar$. The change of the orientation angle ξ from

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90° to 0° lowers the height of the barrier and shifts the position of the minimum towards larger radii. Increasing ℓ the attractive pocket tends to disappear due to the enhancement of the centrifugal barrier. One should note that this is only a static picture. Actually, as the nuclei approach each other, due to the coupling between the relative motion and the internal degrees of freedom, the potential oscillates according to the variation of the orbital angular momentum ℓ and the angle ξ. In this sense the potential under investigation is more complicated than the one of the 3-disks problem [4] and at the same time very realistic. In fact potentials of the type considered here are commonly used - with different units - in atomic and molecular physics.

In fig.3 we show, for a fixed total angular momentum L, the final value of the scattering angle ϕ_f as a function of the initial rotor orientation Φ_i. The initial value of ϕ_i was always set equal to zero (then $\Phi_i = \xi_i$), while the rotor was considered always at rest $I_i = 0$ (then $\ell_i = L$). This choice has been kept through all the calculations presented here. The different trajectories were obtained varying the initial angle $\Phi_i = \xi_i$ from 0° to 180° and taking into account 1000 trajectories for each of the three different values of energy shown in fig.3. Below the Coulomb barrier - $V_B \sim 26.5 MeV$ - (bottom panel) we have a regular and smooth behaviour, while wild fluctuations show up as soon as the energy is increased (middle panel). These fluctuations tend to vanish and give again a regular motion with only a few singularities as the energy is further increased. In fig. 4 we show the deflection function, i.e. the final scattering angle as a function of the total angular momentum. In this case the orientation angle was fixed to the initial value $\Phi_i = \xi_i = 0^o$ while the total angular momentum was varied. Part (a) of the figure shows strong oscillations of the deflection function in between regular regions. Two successive blow-ups (b) and (c) illustrate the persistence of these fluctuations at smaller scales with a very similar structure. This is the typical manifestation of chaos in scattering processes [1–6]: an infinity of singularities having a fractal pattern shows up. Figures 3 and 4 prove that for the heavy–ion system $^{28}Si + ^{24}Mg$ the scattering is chaotic just above the Coulomb barrier. Only above the barrier scattering trajectories can probe the chaoticity of the internal zone. In order to illustrate the dynamics

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inside the pocket we can study the evolution of bound phase space trajectories. In fig.5 we display a Poincaré surface of section for ten confined orbits changing the deformation parameter α. While for $\alpha = 0.1\alpha_{20}$ the motion is completely regular, when α is equal to the value α_{20} corresponding to the deformed nucleus ^{24}Mg one has a real chaotic dynamics. The first KAM tori start to break around $\alpha = 0.15\alpha_{20}$. A magnification of the middle panel of fig.5 is displayed in fig.6 where 90 trajectories are considered.

The system $^{28}Si + {}^{24}Mg$ has no special characteristics and in fact we will show in the following that irregular scattering is rather typical for light heavy-ions, i.e. nuclei whose atomic mass number A lies in the range between A=4 and A=60. In figs.7-9, the final rotor spin I (in units of the maximum spin $I_{max} = \frac{E}{2\Im}$) , the final scattering angle ϕ_f and the reaction time T_f are shown as a function of the initial rotor orientation for the systems $^4He + {}^{24}Mg$, $^{12}C + {}^{24}Mg$, $^{86}Kr + {}^{24}Mg$. The reaction time is defined as the time the system takes to go from an initial asymptotic distance to the final asymptotic one passing through the interaction region. Both initial and final distances are set equal to r=18 fm. A cut-off time equal to T=10^4 fm/c is chosen for those trajectories which remain trapped inside the nuclear pocket. The same fluctuations found for $^{28}Si + {}^{24}Mg$ and characterizing chaotic scattering are evident for these systems as well. Different values of energy and angular momentum are considered to show that chaotic features are not present only in a limited region.

On the contrary for the system $^{86}Kr + {}^{152}Sm$ only a regular motion of the kind shown in fig.10 is found by changing both E and L. This different behaviour has two main reasons. First, as the atomic number of the nuclei increases the enhanced Coulomb repulsion reduces the attractive nuclear pocket. Second, the greater are both the mass of the nuclei and the moment of inertia the slower is the motion of the barrier. The relative motion becomes faster than the one of the internal degrees of freedom, whose slow variation is not able to raise the barrier and trap the spherical nucleus. Therefore, in order to have chaotic motion the two characteristic time scales should be comparable.

Chaotic scattering is not peculiar of the simple 2-dimensional model. In fact taking into

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account the more general 3-dimensional Hamiltonian (1) the possibility for the scattering to be chaotic can even increase. This is shown in fig.11, where the results obtained solving the equations of motion corresponding to the Hamiltonian (1) are displayed for the reaction $^{12}C + ^{24}Mg$. In this case the symmetry axis of the deformed target does not lie completely on the x-y plane, being $\Theta_i = 89^o$. The reaction does not occur on the plane and, in contrast to the previous cases, the angle θ is not constant as a function of time. In particular in correspondence of the irregular regions the final θ-values can be very different from the initial one, see fig.11. In general, when solving the 3D equations, if the planar symmetry is initially assumed it is also maintained throughout the reaction. In this sense the 2D scattering is a particular case of the more general 3D model. However, if a small initial symmetry-breaking occurs - as in the case shown in fig.11 - then the system explores the complete 10D phase space. In the case shown in fig.11 the number of trapped trajectories is greater than in the planar case of fig.8. This is not true in general.

All these features will be studied more quantitatively in the next section.

III. QUANTIFYING CHAOS

After this qualitative introduction which illustrates the ubiquity of chaos in light heavy–ions, in this section we take into account the system $^{28}Si + ^{24}Mg$ as a typical example and we investigate chaotic scattering in a quantitative and detailed way. Possible differences between the 2D and the 3D case are also investigated and discussed.

In fig.12 the final scattering angle ϕ_f as a function of the initial rotor orientation Φ_i is shown for four different small intervals, of initial conditions, i.e. $\Delta\Phi_i = 10^o, 1^o, 0.1^o, 0.01^o$. The planar scattering - panels (a)-(d) - are shown in comparison with the three dimensional case - panels (e)-(h). In this example the total angular momentum is zero and the total energy is 25 MeV. No clear distinction is qualitatively evident in the two cases, nor the successive blow-ups do reveal any deeper difference in the underlying structure.

In order to study possible quantitative differences, let us calculate the fractal dimension

of the repeller. From the final scattering angle reported in fig.12 one can construct the classical cross section counting the number of final angles which fall inside bins of finite size. This cross section as explained in ref. [7] presents very many peaks in correspondence of the (rainbow) singularities which exist around the extrema of the small regular regions. It can be shown [7] that the fractal dimension of the rainbows distribution is equal to the fractal dimension of the repeller. To calculate this fractal dimension the sandbox method has been used as suggested in ref. [7,27]. We use sets of 10^4 trajectories to evaluate the classical cross section $P(\phi_f)$ for the final scattering angle. As a typical example we show in fig.13 (a) the one obtained for the planar scattering of fig. 12(c). The sandbox method consists in counting the number of angles N entering into circles of diameter R, using as centers the most pronounced peaks. The average of $1/N(R)$ over the several centers adopted should scale like R^{-D}, where D is the fractal dimension. This method has been proved to be more efficient than the box-counting technique [27], but the result gives often an estimate which is slightly larger than the true fractal dimension [7]. In fig.13 (b) we plot the points obtained with the sandbox method for the cross section shown in fig.13 (a). In this case the number of rainbows used as centers is 107 and the bin size for the angle is $\Delta\phi_f = 0.1^o$. The points follow a straight line over almost 4 decades showing small deviations only for large R. A fit of the slope gives a fractal dimension D=0.73. To check the accuracy of this value we have also calculated the uncertainty dimension according to ref. [28]. This should be less or equal than the real fractal dimension, and therefore should give us a minimum value. The method of ref. [28] in this case consists in calculating for a fixed uncertainty ϵ the quantity $\Delta T(x_o, \epsilon) = |T(x_0) - T(x_0 + \epsilon)|$, where T is the reaction time corresponding to an initial condition x_0 randomly chosen. If $\Delta T(x_o, \epsilon)$ is greater than a fixed small quantity (we used 50 fm/c in our case, but the actual value is not important) then one says that x_0 is ϵ uncertain. The probability $f(\epsilon)$ to obtain an initial value which is ϵ uncertain - approximated by the ratio between the number of random calls which are uncertain and the total number of calls (10^3 in our case) - should scale like ϵ^{1-D}. Therefore plotting $-Log_{10}f(\epsilon)/\epsilon$ versus $Log_{10}\epsilon$, one gets a line whose slope is D. In order to distinguish between the two methods

we indicate the corresponding values with a superscript u for *uncertainty* and s for *sandbox*. An example of the determination of D^u is given in fig.14 for the same case shown in fig.13. Also in this case the points follow very nicely a straight line over almost 4 decades. The value of the uncertainty dimension is in this case D^u=0.84.

The analysis described above has been done both for the 2D and the 3D scattering for those small intervals of initial conditions considered in fig.12. In all the cases considered a behaviour similar to that shown in figs.13 and 14 is found. The results are summarized in tables 2 and 3. In general the sandbox dimension is not equal to the uncertainty dimension and their relative difference is not always the same. This is probably due to the numerical accuracy, being these kind of calculations very delicate. In order to obtain a value of fractal dimension closer to the real one we have taken the average between the two estimates D^u and D^s. This value is indicated as $\overline{D}$ in the above mentioned tables. If the scattering is fully chaotic - hyperbolic scattering - an exponential law is expected [4] for the reaction time distribution probability $P(T)$. In this case the repeller is characterized by the escape rate Γ defined by the formula $P(T) \sim exp(-\Gamma T)$. In tables 2 and 3 it is also reported the values of Γ extracted by the exponential fits of the time probability distributions displayed in fig.15. In this figure it is shown the logarithm of the collision time probability as a function of collision time for the small intervals of initial conditions into examination. For very long reaction times an exponential law fits very well the distributions, though in some cases strong peaks are evident.

When the scattering is nonhyperbolic, i.e. there is a coexistence of KAM surfaces and chaotic regions [4,28,29], a power law is predicted $P(T) \sim T^{-z}$. It is a fact that all the time probability distributions of fig.15 can be fitted as well adopting a power law. The values of z obtained are reported in tables 2 and 3. The comparison between the two kinds of fit are shown in fig.16 for a typical case. This fact supports the conjecture that our system shows both hyperbolic and nonhyperbolic features. Another indication going in the same direction is the trend of the fractal dimension. As claimed in [28], for hyperbolic scattering the fractal dimension should be the same when going to smaller and smaller intervals. This

does not seem to be always true for the values obtained, see tables 2 and 3. A clearer answer to this question could be probably found by looking for invariant tori or surfaces by means of Poincaré sections. A detailed study along this line is left for the future.

From $\overline{D}$ and Γ one can calculate the average Lyapunov exponent $\overline{\lambda}$ according to the formula $\overline{\lambda} = \Gamma/(1 - \overline{D})$ valid for chaotic repellers [30]. These average exponents are also listed in the tables 2 and 3. Their variations, though limited in a small range and reflecting the fluctuations of $\overline{D}$, indicate that the degree of chaoticity can depend on the specific intervals taken into account. Since the scattering is not exactly hyperbolic, the Lyapunov exponents thus obtained should be compared with those calculated by means of the more standard formula

$$\lambda_\infty = \lim_{d_0 \to 0} \lim_{T \to \infty} \lambda(T) \, , \tag{15}$$

with $\lambda(T)$ defined by

$$\lambda(T) = \frac{\log(d(t)/d_0)}{T} \, . \tag{16}$$

The quantities $d(t)$ and d_0 are the distance between two close trajectories at time T and at time $T = 0$ respectively. In fig.17 it is shown the behaviour of $\lambda(T)$ as a function of time for ten trajectories chosen randomly in the smallest interval of initial conditions considered, i.e. $\Delta\Phi_i = 0.01^o$. The calculations were done both in the 2D, part (a), and in the 3D, part (b), case. In the figure, $\lambda(T)$ shows a clear tendency to have an asymptotic limit λ_∞. The average $\overline{\lambda}_\infty$ over the trajectories considered is reported in the two cases. These Lyapunov exponents are very close to those $\overline{\lambda}$ values obtained previously. However it should be noted that the values of $\overline{\lambda}$ listed in the tables are probably a better estimate of the average degree of chaoticity in the specific interval. In fact both $\overline{D}$ and Γ have been calculated by sampling 10^4 trajectories [31].

In conclusion a detailed investigation of classical heavy–ion scattering reveals a dynamics which seems to have both hyperbolic and nonhyperbolic features and, in contrast to very schematic models, a greater complexity of the flow in phase space which is typical of more realistic potentials [10].

IV. QUANTUM SCATTERING

In the following we discuss the quantal analog of chaotic scattering between a spherical and a deformed nucleus. There is a general agreement to call *quantum chaos* the quantum counterpart of those classical dynamical systems which show chaotic motion. It has been found that quantum chaos presents a behaviour which is different from the quantum counterpart of integrable systems [32–34]. The quantum analog of classical scattering is given by the solution of the Schrödinger equation. In nuclear physics one usually introduces the deformation degrees of freedom as excited quantum states belonging to rotational bands. These states are coupled among each other and these couplings influence the quantum-mechanical evolution of the reaction. This method is often referred to as the coupled-channels approach.

We assume for simplicity and only for the moment that the reaction occurs on a plane. More specifically, considering planar geometry the Schrödinger equation can be written as

$$\left[\frac{-\hbar^2 \Delta^{(2)}}{2m} + \frac{\ell^2 \hbar^2}{2mr^2} + \frac{I^2 \hbar^2}{2\Im} + V(r,\xi) - E\right] \Psi(r,\xi,\phi) = 0\,, \tag{17}$$

where as before m is the reduced mass, ξ is the rotor orientation angle, while $\Delta^{(2)}$ indicates the Laplacian in two dimensions.

From eq.(17), after elimination of the angular dependence of the wave functions [13], one gets the radial coupled–channels equations

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{\ell^2}{r^2} + k_{L-\ell}^2(r)\right]\psi_\ell^L(r) - \frac{2m}{\hbar^2}\sum_{\ell'\neq\ell} V_{\ell'-\ell}(r)\,\psi_{\ell'}^L(r) = 0 \tag{18}$$

with

$$k_{L-\ell}^2(r) = \frac{2m}{\hbar^2}(E - E_{rot} - V_o(r)) \tag{19}$$

$$E_{rot} = \frac{I^2 \hbar^2}{2\Im} \tag{20}$$

$$V_{\ell'-\ell}(r) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(\ell'-\ell)\xi}\, V(r,\xi)\, d\xi \tag{21}$$

$$V(r, \xi) = V_{coul} + V_{nucl} \ . \tag{22}$$

The moment of inertia $\Im$ as well as the ion–ion potential $V(r,\xi)$ are the same considered in the classical case. To simplify the calculation the Coulomb tail of the interaction has been taken away. This hardly influences the scattering which is characterized mainly by the nuclear interaction. In eq.(21) the coupling is taken to all orders and only between the nearest neighbours. For more details cfr. refs. [13,14]. In principle the summation in eqs.(18) should include an infinite number of channels, in practice one considers only the most important N channels. Each of the N eqs.(18) was integrated numerically from the most internal turning point up to an asymptotic distance. There the wave function is the free outgoing solution - a Hankel function in this case - times a coefficient which represents the S–matrix element $S^L_{I,I'}$ for the particular entrance and outgoing channels (I and I' respectively) considered. Solving then the system of equations thus obtained for different channels, one can construct the complete S–matrix.

Let us consider again as a typical example the reaction $^{28}Si +^{24} Mg$. Fig.18 shows the elastic ($I' = 0$) and two inelastic ($I' = \pm2$ $\hbar$) transition probabilities $|S^L_{I,I'}(E)|^2$, calculated at total angular momentum $L = 15\,\hbar$, as a function of incident energy. For an initial spin $I = 0\,\hbar$, 11 final channels were considered, $I = 0, \pm2, \pm4, \pm6, \pm8, \pm10\,\hbar$. An energy step of 20 KeV was adopted for the calculations.

The S-matrix elements show rapid oscillations as a function of energy with a width ranging between 50 and 400 KeV, implying the occurrence of long-living intermediate states of the dinuclear system. Resonances exhibit larger widths, until their complete disappearance, as the energy is increased. A reduced energy step does not reveal any further structures. In ref. [12,13] it was shown that fluctuations manifest themselves only in the region of energy and angular momentum where classical chaos shows up, that is around the potential barrier. In this sense we can say that this irregular behaviour is the manifestation of quantum chaos.

Another indication which gives support to this claim is the fact that the appearance of sharp and grouped structures depends in a sensitive way on the strength of the coupling

term. In ref. [14] it was demonstrated that, by decreasing the strength, fluctuations rarefy and then they disappear completely. On the contrary, an increase of the coupling produces an enlargement of the energy region where fluctuations are present. In the example of fig.18 the coupling term adopted comes out of eqs.(16-20) taking into account the same potential used in the classical chaotic dynamics.

A quantitative study of the fluctuations shown in fig.18 can be obtained by means of autocorrelation function analysis [3,6]. In our case the autocorrelation function for the S-matrix elements can be defined as

$$C_{I,I'}(\epsilon) = < S^*_{I,I'}(E) \, S_{I,I'}(E + \epsilon) > ,\tag{23}$$

where $< >$ denotes the average over an appropriate energy interval ΔE.

For the most populated exit channels, $I' = 0, \pm 2 \, \hbar$, the modulus square of the autocorrelation function obtained from the analysis of the fluctuations is reported in fig.19 (open squares). An energy interval ΔE =4 MeV is adopted to perform the averages and the smooth S-matrix part on this interval is subtracted. The autocorrelation functions present a lorentzian-like behaviour which reminds of the Ericson's fluctuations theory [35–38] developed in the 60s for compound nucleus reactions. The connection is discussed in the next sections. In the figure, lorentzian fits (solid curve) allow to extract a coherence length Γ_{quan}, also shown, that tells us the energy interval in which the S-matrix is correlated with itself. The value of Γ_{quan} varies from 80 to 200 KeV. In the elastic channel, $I' = 0 \, \hbar$, although some deviations from a lorentzian shape are evident, one extracts a coherence length which is almost a factor of 2 bigger than the one corresponding to the inelastic channels.

As discussed also in section VII it is not clear to what extent one can link the properties of the S-matrix fluctuations found here to the well-known features of Random Matrix Theory and Ericson's theory as was done in ref. [3,6,40]. Due to the limited number of resonances one is not able to study their distribution. However, it is usually claimed that RMT predictions apply only to the universal aspect of chaos, while in our case specific characteristics of the system are not neglected and can interfere with the universal ones.

V. SEMICLASSICAL CONSIDERATIONS

In order to link quantitatively the classical dynamics with the quantal counterpart one should perform a semiclassical analysis. In this section we perform this investigation even though in our case we are not allowed to use a priori the semiclassical approximation.

It has been demonstrated [3,6] that, in the semiclassical limit, one can calculate a semi-classical coherence length Γ_{cl} considering the classical distributions of delay time. The latter is defined as the time the system spends in the interaction region.

In fig.20 we show, for the system $^{28}Si + ^{24}Mg$ the delay time distributions calculated in the classical case for 10^4 trajectories. The energies considered are E=28,28.5,29 MeV and the total angular momentum is L=15 $\hbar$. The collision times corresponding to the trivial reflection from the outer barrier (600-1200 fm/c) were taken away in order to obtain the real delay time distribution.

A Fourier transform of $P(T)$ allows to calculate the autocorrelation function, which is given by

$$C(\epsilon) = \left| \int P(T) e^{i\epsilon T} dT \right|^2 \ . \tag{24}$$

According to eq.(24), if the time distribution is exponential $P(T) \sim \exp(-\Gamma_{cl} T)$ then the autocorrelation function is a lorentzian $C(\epsilon) = C(0)/(1 + \epsilon^2/\Gamma^2)$ with width Γ.

In fig.20 (c,d,e) the autocorrelation functions correspondent to the classical delay time distributions of fig.20 (a,b,c) are shown as open squares. A lorentzian fit is also reported as full curve. Small deviations from the lorentzian curve are present as oscillations in fig.20 (e) and can be attributed to the fact that $P(T)$ presents fine structures superimposed on an exponential background. This is in general true as it will be discussed later.

Considering again the lorentzian fits one can extract the widths Γ_{cl} =100,130,150 KeV corresponding to the energies E=28,28.5,29 MeV. The widths reported above are slightly smaller than those in ref. [41] due to the elimination of the first peaks in the reaction time distributions. If one takes an average of the semiclassical coherence length over the same interval of incident energy chosen in the quantum case, the value $\overline{\Gamma}_{cl}$ =250 KeV is obtained.

Comparing then the quantal and the semiclassical values, it turns out that, as in ref. [3,6] Γ_{cl} and Γ_{quan} are equal within a factor of two. This nice agreement links quantitatively classical chaos with its quantum counterpart. At the same time it represents a surprising result since in our problem only the lowest channels are excited and therefore in principle the semiclassical approximation should not work for the rotational degrees of freedom.

Autocorrelation function deviations from a lorentzian curve are present both in the classical and quantal case. In order to understand these deviations let us consider a simulated delay time distribution which mimics the peaks present in fig.20 (b). This is obtained by summing an exponential term plus a gaussian peak

$$F(T) = A \ e^{-\Gamma T} \ + \ B \ e^{\frac{-(T-T_o)}{2\sigma^2}} \tag{25}$$

with A and B constant quantities, see fig.21 (a). The corresponding Fourier transform $f(\epsilon)$ is

$$f(\epsilon) = \ A \ \frac{1}{\Gamma - i\epsilon} + \ B \ e^{i\epsilon T_o} e^{\frac{\epsilon^2 \sigma^2}{2}} \ . \tag{26}$$

Then the autocorrelation function $C(\epsilon)$ is given by the modulus square of $f(\epsilon)$. The phase in front of the second term of eq. (25) gives rise to oscillations in $C(\epsilon)$ whose magnitude depends on the height of the gaussian peak with respect to the exponential background. This is illustrated in fig.21 (c) where the analytical $C(\epsilon)$ (full curve) is plotted in comparison with the numerical evaluation of the Fourier transform, open squares. Adding other gaussian peaks of the kind $C \ e^{\frac{-(T-T_i)}{2\sigma^2}}$ to eq. (25) one should add other terms of the kind $C \ e^{i\epsilon T_i} e^{\frac{\epsilon^2 \sigma^2}{2}}$ to eq. (26). Thus other phases enters into $f(\epsilon)$ and the behaviour of $C(\epsilon)$ becomes even more complicated. This is illustrated in fig.21 (b,d), where two gaussians peaks are considered. In general the analytical formula is very well reproduced by the numerical calculation of $C(\epsilon)$, open squares. The dashed lorentzian curves shown in figs. 18(c,d) are the $C(\epsilon)$ corresponding to the exponential used for the simulated $P(T)$ which in this case has $\Gamma = 130 \ KeV$. The width of the lorentzian is modified by the oscillations and its value depends on the position and magnitude of the peaks of $P(T)$. Note that to obtain $C(\epsilon)$ in MeV we have divided the time expressed in fm/c by $\hbar c$.

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The above considerations are important since as shown in figs. 13 and 17 very often one has peaks superimposed on an exponential-like behaviour for the delay time probability. These peaks are due to the small quasi-regular regions inside the chaotic sea which give a greater contribution to the probability. However they cannot be easily separated and they influence inevitably also the behaviour of the most chaotic trajectories. In other words the interplay of different reaction times, due to the strong mixing between regular and irregular motion, is in general difficult to disentangle completely in such a complex reaction mechanism.

In general the claim often advanced that the lorentzian distortions of $C(\epsilon)$ are only due to finite size effect [42] is not always true.

VI. REALISTIC CALCULATIONS

The quantal approach we have used up to now is a simplified description of heavy ion scattering. A more realistic model should take into account: a) a three-dimensional description of the scattering; b) the effect of other degrees of freedom (like vibrations or nucleon transfer) by means of an absorption term in the potential; c) the calculation of cross sections, angular distributions and other observables directly comparable with experimental data.

The role of absorption was studied in ref. [12]. Adding an imaginary component to the interaction it was demonstrated that when the absorption at the barrier is strong enough the fluctuations in the transition probabilities can be completely washed out. As it is discussed in the next section, an important feature which has been found experimentally in the heavy ion reactions of the kind investigated here, is the superficial transparency of the potential. Thus in our case the assumption of a weak absorption is a very realistic approximation. Semiclassically this means that long lived trajectories give an appreciable contribution.

In ref. [13,14] it has been shown that fluctuations in the transition probabilities are concentrated around the barrier. Due to this reason, increasing the initial orbital angular momentum produces a shift in the energy range where these fluctuations are clustered.

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This is why when one sums over the angular momenta to calculate cross sections [13,14] and angular distributions [43] very complicated and irregular structures appear again as a function of energy. In the following, in order to consider a very realistic quantum description of the reaction between a deformed nucleus and a spherical one, we use the three-dimensional coupled channel code FRESCO [44]. The latter is a sophisticated program which can be considered the quantal analog of the three-dimensional classical picture described in sections II and III. At the same time it gives us the possibility to take into account the three points discussed above.

The elastic transition probability calculated by using FRESCO is shown in fig.22 for the systems $^{28}Si + \, ^{24}Mg$ and $^{12}C + \, ^{24}Mg$. The total angular momentum considered is L=10 $\hbar$. The real part of the nuclear ion-ion potential is the same used for the 2D calculations. The tail of the Coulomb interaction is taken properly into account. A small absorption is considered taking an imaginary potential W which has a Woods-Saxon shape: $W(r) = W_0/[1 + exp((r - r_0)/a)]$, with W_0=0.2 MeV, r_0=0.86 $(A_1^{1/3} + A_2^{1/3})$ fm and a=0.2 fm. The parameters used are close to those adopted in ref. [45] for a similar system. We have considered only two rotational states, the 2^+ (at E=1.26 MeV) and the 4^+ (E=4.21 MeV) in the deformed nucleus plus the ground state 0^+. Only a coupling between the nearest neighbours is considered. The total number of exit channels is 9 [14]. The coupling factor is again given by eq.(21).

In fig.22 oscillations similar to those present in the simpler 2D calculation of fig.18 are shown. The energy step (in the center of mass frame) used is 0.046 MeV and 0.053 MeV for $^{28}Si + \, ^{24}Mg$ and $^{12}C + \, ^{24}Mg$ respectively. In this case, due to the absorption considered, we have less structures and they have a smoother behaviour than in the 2D calculations presented here where the absorption was neglected. However, when a summation over the angular momenta is performed and the cross section at backward angles is calculated, very complicated fluctuations appear. Examples of excitation function are displayed in figs.23 and 24 for the same systems.

As found in the planar case [14] (not shown here for lack of space), cross sections fluctuate

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irregularly for energies greater than the barrier in all the channels considered. In general no qualitative changes are found in going from 2D to 3D. However, in the latter case, the cross sections oscillate on a larger scale.

In order to study in a quantitative way these fluctuations, we divide each point of the cross section by its local average value. That is we consider the quantity $X(E) = \frac{d\sigma/d\Omega}{<d\sigma/d\Omega>}$. This local average should be taken over an interval ΔE which is bigger than the average width of the structures and much smaller than the full energy range considered. This procedure eliminates the smooth behaviour of the cross section and at the same time enables one to investigate the fluctuations of a quantity which is dimensionless and vary over a few units [47]. The fluctuations thus obtained are shown in fig.25 for the system $^{28}Si + ^{24}Mg$. The actual local average is done over an energy interval $\Delta E = 0.8$ MeV.

One can now proceed in evaluating the autocorrelation functions. In this case, we adopt the standard formula used for cross sections [46,47]

$$\frac{< X(E)\ X(E+\epsilon) >}{< X(E) >\ \ < X(E+\epsilon) >} - 1 \ . \tag{27}$$

These autocorrelation functions are displayed in fig.26 for the states considered in fig.25. The dashed curves are lorentzians whose widths are also reported in the figure. The autocorrelation functions displayed in fig.26 are different from those shown in fig.19. The latter go to zero and follow more closely a lorentzian shape, even though the widths of the lorentzians for the inelastic channels are comparable. However the meaning of $C(\epsilon)$ in the two case is deeply different. In fact the one shown in fig.19 refers to the S-matrix for a fixed orbital angular momentum (equal to the total one since I=0) while the other correspond to the cross section obtained summing over many (60 in this case) ℓ. In heavy–ion scattering, due to the large size of the nuclei, a great number of waves contribute. Therefore any realistic calculation should take it into account a sum over numerous ℓ-waves.

Another quantity which can be determined experimentally is the angular distribution, i.e the differential cross section as a function of the detection angle for a fixed energy. By means of the code FRESCO we have calculated elastic and inelastic angular distributions

as a function of the incident energy. It is found a strong oscillating behaviour as the energy is above the barrier and for backward angles. In general the angular distribution at large angles is dominated by the nuclear interaction, while the Coulomb one predominates at forward angles. Then the backward angles fluctuations are strictly connected to the internal part of the interaction which classically shows a chaotic dynamics.

In fig. 27 the elastic angular distributions as a function of incident energy are shown. Only the angles in the range with $86^o < \theta_{cm} < 178^o$ are plotted in order to outline the irregular behaviour. similar features were found in ref. [43] for the 2D quantal model.

In conclusions, we have demonstrated that no drastic change appear in the qualitative features of the scattering in passing from 2D to 3D. An irregular behaviour in cross section and angular distributions persist and can be connected to the underlying chaotic classical scattering. In the next section we review the main experimental features of heavy-ion scattering around the Coulomb barrier for nuclei of the kind considered here.

VII. EXPERIMENTAL OVERVIEW AND DISCUSSION

In nuclear physics cross section fluctuations have been observed since the 60s, when nucleon-nucleus reactions started to be intensively studied [48]. Predicted by Ericson [35–38], fluctuations in compound nucleus cross sections were detected [49–51] at excitation energies above the neutron evaporation barrier. That is in the energy region of strong overlapping resonances, where the level spacing D is very small in comparison with the level width Γ, $\Gamma/D \gg 1$. Fluctuations are generated by the random action of the very many intermediate levels which connect the entrance and the exit channels. According to Ericson's theory, autocorrelation functions of experimental data have a lorentzian shape whose width Γ, the coherence length, gives the energy range within which the intermediate levels are excited coherently. Therefore Γ represents the average level width of the intermediate compound nucleus and gives information on the average lifetime of the compound nucleus $\tau = \hbar/\Gamma$ and on the level density. Fluctuations have a statistical nature, but are experimentally

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reproducible. However, experiments with heavier projectiles - performed almost at the same time - revealed excitation function fluctuations with different features. The first system to be studied was $^{12}C + ^{12}C$ [52]. In this case fluctuations started around the Coulomb barrier presenting structures with widths of different sizes. In general these structures, which were present in several reaction channels, became broader as the incident energy increased. The coherence lengths extracted from these experiments were larger than those previously found in nucleon-nucleus scattering - 100-300 KeV against 10-50 KeV- and correlation analyses showed a nonstatistical origin. Similar characteristics were observed for $^{12}C + ^{16}O$, and $^{16}O + ^{16}O$ among several other systems [48]. Due to the peripheral kind of these reactions and the unusual strongly attractive nucleus-nucleus potential at large distances, it was postulated that these oscillating structures should have a molecular-like nature substantially different from that of the average compound nucleus.

Going to heavier systems, a more complex behaviour was detected. In correspondence of excitation function fluctuations, anomalous large and highly oscillating angular distributions were observed. Typical examples of this behaviour are the systems $^{16}O + ^{28}Si$ [53] and $^{12}C + ^{28}Si, ^{32}S$ [54], where these features were first observed. Again a dinuclear molecule was thought to be the origin, but the mechanism soon appeared much more complicated: systems leading to the same nuclear composite showed different structures; it was not always possible to understand the angular distributions in terms of only one single wave, on the contrary several angular momenta around the grazing value were involved [55]. The phenomenon has been intensively studied and, as in the case of Ericson's fluctuations, a vast literature can be found on the subject. Fundamental review papers, both on the many experiments performed and the theoretical models proposed to explain heavy–ion resonances, are those of Erb and Bromley [48] and Braun-Munzinger and Barrette [56]. They say clearly that fluctuating phenomena in light systems seem to have a common nature: there are only quantitative, but not qualitative differences from system to system. However, notwithstanding the great effort spent during these years, there is not yet a quantitative theoretical understanding of this behaviour: all the advanced models fail - partly or completely - in reproducing the large

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set of existing data. The only model-independent consideration which comes out naturally from the experimental analysis is the unexpected presence of a very weak surface absorption. In other words, a relatively small number of channels is involved.

Though the interest in these intriguing phenomena diminished in the 80s, some groups have continued the experimental research. Thus fluctuations were recently found in the elastic and inelastic cross sections of heavier nuclei like $^{28}Si + ^{28}Si$ [57], $^{24}Mg + ^{24}Mg$ [58] and $^{24}Mg + ^{28}Si$ [59]. At the same time excitation function fluctuations were observed also in deep inelastic collisions of several systems like $^{19}F + ^{89}Y$ [60], $^{28}Si + ^{64}Ni$, $^{28}Si + ^{48}Ti$ [46,61]. Again, differences from Ericson's theory were found, mainly because correlations between several channels and a clear angular dependence were evidenced. In ref. [47] cross section fluctuations were measured for several windows of energy loss, establishing this way a connection between oscillating phenomena in elastic and damped reactions.

The connection between fluctuations in light heavy–ion reaction and a chaotic mechanism though generically addressed already in ref. [3] was stressed for the first time in the conclusions of ref. [47]. In sections II-VI of this paper we have presented a model which exhibits chaotic scattering and is able to reproduce in a semiquantitative way the experimental phenomenology for light heavy–ion reaction discussed above. The puzzling irregularities observed experimentally find a natural explanation in the framework of chaotic scattering considering only a few degrees of freedom.

One could think that having used rotational states chaotic scattering is limited only to this kind of excitations. Actually, features very similar to those here discussed have been found both classically [9] and quantum-mechanically [15] for heavy–ion reactions considering vibration modes.

We can therefore conclude that irregular scattering has a well established theoretical and experimental foundation in light heavy-ion collisions.

In general the single fluctuations are not theoretically reproducible - quantum chaos seems to maintain a strong sensitivity on the input parameters - and one should compare instead autocorrelation functions, widths distributions or other statistical quantities. However

absorption can help in increasing the theoretical predictive power smearing out the wildest fluctuations.

Ericson fluctuations are often claimed to be the quantum manifestation of classical chaotic scattering. Actually Ericson's theory was proposed for compound nucleus reactions and considers only the universal statistical aspects of chaotic scattering moreover, as already mentioned, it applies only for strongly overlapping resonances. In this sense it has been related to the Random Matrix Theory [3,6,40]. In our approach direct, semidirect and long lived reactions are taken explicitly into account. Partially broken invariant surfaces which correspond to what is usually called soft chaos [32] seem to be present. Moreover the fact that in our case $\Gamma/D \leq 1$ indicates a regime of chaoticity produced by a dynamical mechanism which differs from the Ericson's one. We want to stress that both regular and fully chaotic scattering are two extreme exceptional cases. In general one finds more often a mixed situation which lies in between. This situation is the most complicated to deal with, expecially in the quantum case where the chaos-to-order transition is more elusive. In this respect a lot of work has still to be done in order to characterize quantitatively this transition.

VIII. SUMMARY

It has been shown that chaotic scattering represents a real possibility in collisions between light heavy ions and that it can explain the irregular fluctuations observed experimentally. A few degrees of freedom can generate a very complicated and unpredictable motion expecially when semiclassical approximations are used. This is an important result both for nuclear physics and for more fundamental questions like the existence and the features of quantum chaos. These investigations allow to reinterpret standard approaches - although for the moment only in a generic way - in the new framework of the transition from order to chaos. The study of heavy–ion scattering is particularly interesting due to its privileged position between the classical and the quantum world.

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TABLES

TABLE I. Parameters used for the deformed nuclei studied in the text.

nucleus	α_{20}	Q_o	$\Im\hbar^{-2}$
		(fm^2)	(MeV^{-1})
^{24}Mg	0.42	57.	2.378
^{252}Sm	0.246	360.	25.

TABLE II. Characteristic quantities in the case of 2D classical scattering for E= 25 MeV and L=0 $\hbar$, see text.

$\Delta\Phi_i$ (deg)	D^u	D^s	$\overline{D}$	Γ 10^{-3} (c/fm)	$\overline{\lambda}$ 10^{-3} (c/fm)	z
10	0.79	0.85	0.82	0.4	2.20	3.14
1	0.83	0.89	0.86	0.48	3.43	3.33
0.1	0.84	0.73	0.79	0.4	1.88	4.3
0.01	0.91	0.77	0.84	0.37	2.3	4.34

TABLE III. Characteristic quantities in the case of 3D classical scattering for E= 25 MeV, L=0 $\hbar$, $\phi_i = 90^o$ and $\Phi_i = 45^o$. See text.

$\Delta\Phi_i$ (deg)	D^u	D^s	$\overline{D}$	Γ 10^{-3} (c/fm)	$\overline{\lambda}$ 10^{-3} (c/fm)	z
10	0.82	0.83	0.82	0.38	2.10	2.98
1	0.82	0.77	0.79	0.35	1.70	2.7
0.1	0.82	0.73	0.77	0.28	1.23	2.57
0.01	0.91	0.70	0.81	0.33	1.71	3.41

FIGURES

FIG. 1. Coordinate system used. Polar coordinates r, θ and ϕ specify the position of the spherical projectile nucleus, while Θ and Φ are the Euler angles of the intrisic frame of the deformed target nucleus.

FIG. 2. The ion-ion potential adopted is plotted for the system $^{28}Si + ^{24}Mg$. Three value of orbital angular momentum $\ell = 15, 35, 45\ \hbar$ are shown for two orientation angles, i.e. $\xi = 0^o$ (dashed curve), and $\xi = 90^o$ (full curve).

FIG. 3. The final scattering angle ϕ_f is plotted as a function of the initial rotor orientation Φ_i for the reaction $^{28}Si + ^{24}Mg$. For a fixed total angular momentum $L = 15\ \hbar$, three different energy values are considered. See text.

FIG. 4. The deflection function for the same system of fig.3 at E=28 MeV, a). Two successive magnifications are also shown in b) and c). See text.

FIG. 5. Poincaré surfaces of section for 10 trajectories bound inside the interaction zone. The system considered is $^{28}Si + ^{24}Mg$. Different values of the deformation parameter α are used in order to illustrate the order-to-chaos transition in the interaction region. More precisely $\alpha/\alpha_{20} = 1, 0.15, 0.1$ going from top to bottom panel, being α_{20} the real deformation value of ^{24}Mg used in this paper.

FIG. 6. Magnification of the surface of section shown in fig. 5 (middle panel) for $\alpha/\alpha_{20} = 0.15$. In this case 90 trajectories are considered.

FIG. 7. Final values of the rotor spin I_f (in units of the maximum spin $I_{max} = \frac{E}{2\Im}$), the deflection angle ϕ_f and the reaction time T_f as a function of the initial rotor orientation for the 2D scattering of the system $^4He + ^{24}Mg$. The values of the total energy and angular momentum are E=6 MeV and L=5 $\hbar$ respectively.

FIG. 8.   The same of fig.7 for the system $^{12}C + ^{24}Mg$. In this case the total energy and the total angular momentum are E=12 MeV and L=5 $\hbar$ respectively.

FIG. 9.   The same of fig.7 for the system $^{86}Kr + ^{24}Mg$. In this case the total energy and the total angular momentum are E=63 MeV and L=10 $\hbar$ respectively.

FIG. 10.   The same of fig.7 for the system $^{86}Kr + ^{152}Sm$. In this case the total energy and the total angular momentum are E=270 MeV and L=0 $\hbar$ respectively.

FIG. 11. Final values of the rotor spin, the deflection angles (ϕ_f and θ_f) and the reaction time versus the initial rotor orientation for the reaction $^{12}C + ^{24}Mg$ considering 3D scattering. The total energy is E=5 MeV and the total angular momentum is L=12 $\hbar$.

FIG. 12. The final scattering angle ϕ_f is shown as a function of the initial conditions for various small intervals. The reaction is $^{28}Si + ^{24}Mg$ at E=25 MeV and zero total angular momentum. In (a-d) and (e-h) 2D and 3D is considered. In the 3D case the initial orientation angle is $\Phi_i = 45^o$.

FIG. 13.   In the upper part (a) the classical cross section is shown as a function of ϕ_f, for the case illustrated in fig.12(c). In the lower part (b) we plot the quantity $-ln < 1/N(R) >$ versus $lnN(R)$ to calculate the fractal dimension D by means of the sandbox method. In this case one obtains D=0.73, see text.

FIG. 14.   The behaviour of $Log_{10}(f/\epsilon)$ versus $Log_{10}(\epsilon)$ is shown for the same case displayed in fig.13. The slope, which in this case is D=0.84, gives the uncertainty dimension, see text.

FIG. 15. The reaction time probability distributions for the intervals of initial conditions shown in fig.12. The straight lines are linear fits whose slope gives the escape rate Γ reported in tables 2 and 3.

FIG. 16. The reaction time probability distribution for the case shown in fig.12(f) and 15(f) is displayed in comparison with two different fits: an exponential law (a) and a power law (b) are adopted.

FIG. 17.   The function $\lambda(T)$ is shown versus time for ten trajectories randomly chosen in the intervals of initial conditions considered in fig.12(d) and fig.12(h) in order to estimate the Lyapunov exponents. Both in the 2D (a) and in the 3D (b) case an asymptotic value λ_∞ is approached. The value $\overline{\lambda}_\infty$ shown represents the average over the asymptotic limits of the trajectories considered.

FIG. 18.   Quantal elastic transition probability as a function of incident energy. The calculations are the result of the 2D coupled channels approach described in the text. An energy step equal to 20 KeV is used.

FIG. 19.   Autocorrelation functions (open squares) corresponding to the transition probabilities shown in the previous figure. The full curves are lorentzian fits whose width is also reported. See text.

FIG. 20.   Classical delay time probability distributions for the reaction $^{28}Si + {}^{24}Mg$. The scattering is in 2D for L=15 $\hbar$ and E=28,28.5,29 MeV (a,b,c). The respective autocorrelation functions are plotted in (d,e,f) as open squares, while lorentzian fits are drawn as full curves. The widths are also reported.

FIG. 21.   Simulated delay time probability distribution (a,b) and respective autocorrelation functions (c,d). The full curves in (c,d) correspond to the analytical $C(\epsilon)$, while the open squares refer to the numerical evaluation. The dashed curves are lorentzian whose width Γ=130 KeV corresponds to the exponential law used in (a,b), see text.

FIG. 22.   Elastic transition probabilities calculated by means of the 3D code FRESCO (see text) for the systems $^{28}Si + {}^{24}Mg$ (a) and $^{12}C + {}^{24}Mg$ (b). The total angular momentum is L=10 $\hbar$. See text.

FIG. 23.   Excitation functions for the system $^{28}Si + {}^{24}Mg$ obtained by means of the 3D code FRESCO, plotted for $\theta_{cm} = 178^o$. The channels 0^+, 2^+ and 4^+ are displayed. See text for more details.

FIG. 24. The same of fig.23 for the system $^{12}C + ^{24}Mg$ and $\theta_{cm} = 180^o$.

FIG. 25.   The fluctuations found for the excitation functions displayed in fig.23 are evidenced by dividing the cross section $d\sigma/d\Omega$ by its average local value obtained by considering an interval ΔE=0.8 MeV. See text.

FIG. 26. Autocorrelation functions corresponding to the fluctuations displayed in fig.25 (full squares). Lorentzian curves (dashed) are shown for comparison. The widths are also reported.

FIG. 27. Elastic angular distributions, with $86^o < \theta_{cm} < 178^o$, as a funtion of the incident energy. The calculations refer to the reaction $^{28}Si + ^{24}Mg$ and were done by means of the 3D code FRESCO.